



Isaac Newton (1642-1727)

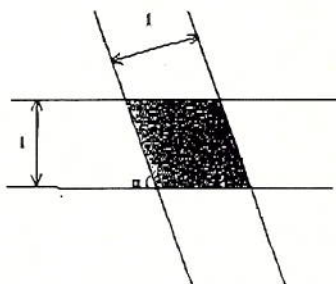
High School Mathematics Contest Individual Exam

1. In an arithmetic sequence the first term is 2, the last term is 29 and the sum of all terms is 155. Find the common difference.
2. Find all real solutions x satisfying

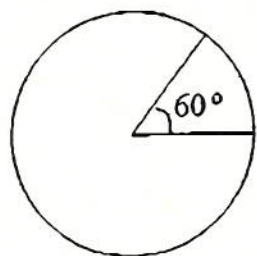
$$3^{2x+2} - 3^{x+3} - 3^x + 3 = 0$$
3. Factor completely (if possible) into its prime factors:

$$x^2 - y^2 - z^2 + 2yz + x + y - z$$
4. If $\log_3 3 = p$ and $\log_3 5 = q$, then, in terms of p and q , find $\log_{10} 5$.
5. If $\frac{4x}{2^{x+y}} = 8$ and $\frac{9^{x+y}}{3^{5y}} = 243$, solve for x and y .
6. Reduce the following expression to its integer value:

$$d = \sqrt{3 + 2\sqrt{2}} - \sqrt{3 - 2\sqrt{2}}$$
7. If Θ is an acute angle and $\sin \frac{\Theta}{2} = \sqrt{\frac{x-1}{2x}}$, find $\tan \Theta$.
8. Two strips of width 1 overlap at an angle of α as shown. Find the area of the overall (shown shaded) in terms of α .

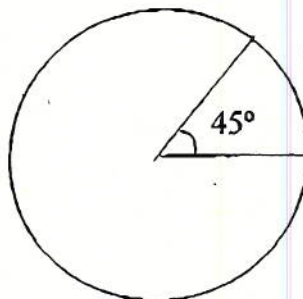


9. If an arc of 60° on circle I has the same length as an arc of 45° on circle II, find the ratio of the area of circle I to that of circle II.



$$S_1 = r_1^2 (\pi/3)$$

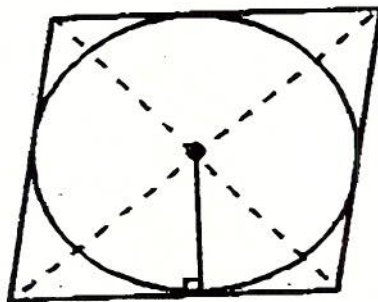
I



$$S_2 = r_2^2 (\pi/4)$$

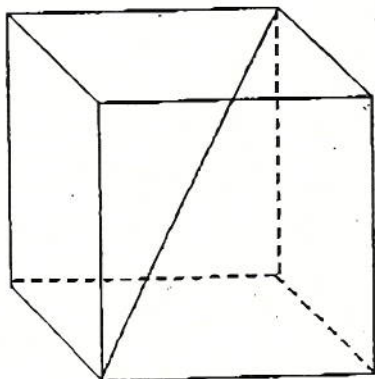
II

10. Find the area of a circle inscribed in a rhombus whose perimeter is 100 and whose longer diagonal has length 40.



11. In the adjoining figure is a diagonal of a cube. If PQ has length a , find the surface area of the cube.

Q



P